Remembering Hunter Snevily

Tanbir Ahmed
Department of Computer Science
and Software Engineering
Concordia University
Montréal, QC H3G 1M8, Canada
ta_ahmed@cs.concordia.ca

André Kézdy
Department of Mathematics
University of Louisville
Louisville, KY 40292 USA
andre.kezdy@louisville.edu

Douglas B. West
Department of Mathematics
Zhejiang Normal University
University of Illinois
Urbana, IL 61801, USA
dwest@math.uiuc.edu

Hunter Snevily: An Original *by Douglas B. West*

With the untimely passing of this still-young mathematician, the world has lost an "original". Hunter Snevily came to the University of Illinois in the mid-1980s and became only my fifth Ph.D. student, completing his thesis in 1991. I was also young then and learned as much from him as he did from me.

Hunter's main work was in extremal set theory, where he made a significant contribution. As a graduate student, he formulated a conjecture (1991) [29] bounding the size of a family of sets under intersection constraints. He conjectured that if $\mathcal{L}$ is a set of k positive integers and $\{A_1, \ldots, A_m\}$ is a family of subsets of an n-set satisfying $|A_i \cap A_j| \in \mathcal{L}$ whenever $i \neq j$, then

$m \leqslant \sum_{i=0}^{k} \binom{n-1}{i}$. Like many of Hunter's conjectures, this was ambitious; it would beautifully unify classical results of de Bruijn and Erdős (1948) [14], Bose (1949) [9], Majumdar (1953) [24], Ryser (1968) [27], Frankl and Füredi (1981) [18], and Frankl and Wilson (1981) [19]. Let us call $\sum_{i=0}^{k} \binom{n-1}{i}$ the *Snevily bound*; note that its value is n when $k = 1$. De Bruijn and Erdős had proved the Snevily bound when $\mathcal{L} = \{1\}$; Bose and Majumdar both extended this by proving it whenever $|\mathcal{L}| = 1$. Frankl and Füredi had earlier conjectured the Snevily bound for the special case $\mathcal{L} = \{1, \ldots, k\}$. In a well-known result using linear independence of polynomials, Frankl and Wilson proved a weaker bound for all $\mathcal{L}$, namely $m \leqslant \sum_{i=0}^{k} \binom{n}{i}$.

In a 1994 paper [31], Hunter proved his bound under the added hypothesis that none of the sets A_i have size in $\mathcal{L}$ or that those with size in $\mathcal{L}$ have a common element. This theorem was used to prove his conjecture when n is sufficiently large and to prove a special case of the Frankl–Füredi Conjecture. In a 1999 paper [34], Hunter proved his conjecture in the case where $\mathcal{L}$ consists of k consecutive integers, generalizing a well-known inequality of Fisher [17] while simplifying and extending an intervening proof by Ramanan [26] of the Frankl–Füredi Conjecture.

Hunter's quest for the proof continued. Through firm conviction and diligent endeavor (see André Kézdy's account for detail), he finally gave an elegant proof (2003) [36] of his conjecture, published in *Combinatorica*. This was his most impressive result, marking the end of a decade-long journey. It is hard to describe his emotions on attaining this goal that he sought for so long; he sent me a poem he wrote to express the depths and heights of the process.

The result was just a beginning for others. The technique built upon the polynomial method: instead of defining a smaller space in which to capture polynomials associated with sets in the family, Hunter added other independent polynomials to the natural space in order to crowd those from the family into a smaller subspace, thereby improving the Frankl–Wilson bound to the Snevily bound. His methods have since been extended to obtain further results, some conjectured by Hunter, in which only the congruence classes of intersection sizes modulo a given prime need to lie in $\mathcal{L}$, or in which the constraints are applied to intersections of t members of the family instead of $t = 2$ (see [10, 11], for example).

Hunter also did important work on several other problems, which others have built on. The well known Chvátal's Conjecture (1974) [12] states that every hereditary family $\mathcal{F}$ of sets has a largest intersecting subfamily consisting of sets with a common element. Schönheim [28] proved this when the maximal members of $\mathcal{F}$ have a common element. Chvátal proved it when there is a linear order on the elements such that $\{b_1, \ldots, b_k\} \in \mathcal{F}$ implies $\{a_1, \ldots, a_k\} \in \mathcal{F}$ when $a_i \leq b_i$ for $1 \leq i \leq k$. A family $\mathcal{F}$ has

x as a *dominant element* if substituting x for any element of a member of $\mathcal{F}$ not containing x yields another member of $\mathcal{F}$. Hunter's 1992 result [30] greatly strengthened both Schönheim's result and Chvátal's result by proving the conjecture for all families having a dominant element; it was the first major advance on the problem. This beautiful argument applied a result of Berge [7] about partitioning hereditary families into disjoint pairs of members. Hunter posed further questions on the topic that can be found in [38]. Borg [8] generalized Hunter's result to a setting with weights on the elements.

In graph theory, Hunter was interested in the snake-in-the-box problem, graph labelings, and graph pebbling. The snake-in-the-box problem seeks the longest induced cycle in the n-dimensional hypercube; Hunter's upper bound in [32] stimulated further upper bounds in [15, 16, 23], but it seems that it still is not known whether there is a constant c less than 1 such that $c2^n$ is an upper bound.

A *graceful labeling* of a graph with m edges is an injection from its vertex set into $\{0,\dots,m\}$ such that the differences between adjacent vertices are the numbers $1,\dots,m$. Hunter developed a technique in [33] for generating a special type of graceful labeling. This well-cited paper stimulated many other constructions of special labelings. Another idea for labeling that he had in his thesis but did not publish led to [20], and he collaborated with Kézdy in [21] on labeling results toward the Graceful Tree Conjecture (that all trees have graceful labelings) using the Combinatorial Nullstellensatz. Hunter was always interested in the application of sophisticated, powerful tools.

Hunter's most-cited paper [25] concerns graph pebbling, joint with Pachter and Voxman. A collection of pebbles is distributed at the vertices of a graph. A *pebbling move* takes two pebbles from one vertex and moves one to a neighbor vertex, losing the other. The question is how many pebbles are needed so that from any initial distribution of those pebbles, a pebble can be moved to any specified vertex. This paper and Hunter's later paper [37] with Foster added to the extant conjectures on the subject and together have been cited in more than 50 papers.

Indeed, one of Hunter's notable qualities was that he was a conjecture-making machine. He spewed out conjectures in many areas: set theory, number theory, graphs, groups, etc. Often the emails were followed up in a day or two by counterexamples he found, but many of the conjectures turned out to be real gems that both students and seasoned researchers could sink their teeth into.

One example became known as Snevily's Conjecture (1999) [35]: Given an abelian group G of odd order, and subsets $\{a_1,\dots,a_k\}$ and $\{b_1,\dots,b_k\}$ of G, there exists a permutation π of $[k]$ such that $a_1+b_{\pi(1)}, a_2+b_{\pi(2)},\dots,a_k+b_{\pi(k)}$ are distinct. Alon (2000) [5] proved this for cyclic groups of prime order. Dasgupta et al. (2001) [13] proved it for all cyclic groups. Finally, after a decade, the conjecture was proved for all groups by a young mathematician Arsovski (2009) [6]. Terence Tao devoted a section to Snevily's Conjecture in his well-known book *Additive Combinatorics.*

Aggressively prodding colleagues to study his conjectures was one of Hunter's techniques in mentoring students. After completing his doctorate at Illinois he went to Caltech for a post-doctoral appointment as a Bateman Instructor for two years, where he was unusually active in mentoring students. Among them was Lior Pachter, now a professor at Berkeley, who described Hunter's teaching and mentoring in a tribute upon Hunter's retirement:

> . . . although I had learned many theorems, I knew that I didn't know how the theorems were discovered in the first place. I still had not had real contact with mathematics – I had not experienced the thrill of conjecture. Hunter's course changed all of that. He is a master of conjecture, and his skill and passion for mathematics spilled over to the class. He taught us graph theory and matroid theory by empowering us to discover ourselves. Classical theorems and cutting edge conjectures shared an equal footing in the class, and we learned how research happens, and how to think.
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> . . . During my undergraduate years, Hunter was much more than a teacher and collaborator to me. He took me to my first math conference, shared his research thoughts and anxieties, and we would talk a lot about life. Perhaps the fact that he was a postdoc at the time, and I an undergrad- uate, meant that our distance in the graph of life was short, and therefore it was possible for us to become friends. I'm very fortunate to have my career launched by such a friend.

I was actually much closer then Pachter to Hunter's age; Hunter was only three years younger than I. As a graduate student in Illinois, he was already more mature in many ways than I, the assistant professor. The passion he brought to mathematics he brought also to his interactions with people. His mathematical tastes were impeccable; he refused to publish results of his own that he did not think were sufficiently elegant. The analogue of this in his life was tremendous integrity, insistence on doing the right thing, and never being afraid to state what he believed. From him I learned much about the proper way to treat graduate students and the proper way to do mathematics. I thank him for that. When I say he was an "original", I mean there are none like him, in so many ways.

In 1993 Hunter moved to the University of Idaho in Moscow, where he remained. It was an ideal setting for his love of the mountains and back country. Nevertheless, the world remained his office, via the internet. He maintained his collaboration with Andé Kézdy in Louisville throughout his life. He sent conjectures for my students to work on during my summer research program, such as in [38]. Even after retirement, he continued to seek out and mentor students, as described by Tanbir Ahmed below. They started a new chapter in Hunter's research, studying number-theoretic aspects of Ramsey theory, such as Schur numbers and van der Waerden numbers, often via experimental computational means.

I learned of Hunter's illness when he sent me an email message with the subject line "park". It contained the diagnosis, in one line, followed by a poignant and humorous story he had written about the interaction between a son and a father (perhaps himself) with Parkinson's disease. Shocked, I apologized for having previously complained about being unable to understand some of his emails about new conjectures. With typical grace, he responded

> hey Doug - don't sweat it
>
> my math career is over but i am SO HAPPY i met you and that u had a course on finite sets
> i put my heart and soul into my thesis conjecture
> it was a thrill to solve it
> it was my Mount Everest
> the hypercube and me were meant for each other
>
> DEF- S_n is size of snake in Q_n (induced cycle)

followed by another conjecture.

Hunter and I wrote only one joint paper. I wish I had put more effort into understanding and working on his conjectures. It always seems that there will be time for such things later after more urgent tasks are completed, but then suddenly there is no more time.

Hunter Snevily: Remembrances *by André Kézdy*

I met Hunter Snevily in the late 1980's when we were graduate students together in the Department of Mathematics at the University of Illinois at Urbana-Champaign. We both studied combinatorics and graph theory under the supervision of Doug West. Hunter was six years older, but I had been in the program longer. Consequently he treated me like an older brother, respectfully, but mostly competing with me. We both enjoyed this friendly rivalry – it propelled our collaboration for a lifetime.

My first impressions of Hunter were slightly alarming. I greatly enjoyed conversations that obviously revealed that he was very familiar with many difficult open problems. I learned about many from him. However he seemed over-confident that he could solve any of these conjectures and brashly outspoken about his ambitions to anyone that would listen. He was the complete opposite of my quiet, circumspect and cautious nature. I often did not know how to temper his enthusiasm for another attack on an impossible conjecture. Over time I came to realize that my first impressions of him were mistaken. All of this apparent bravado was merely the façade of his enthusiasm, optimism and, most importantly, profound belief in the power of creativity. He believed, powerfully, that many difficult conjectures have remained unsolved, not for lack of technical expertise or sheer effort, but because a clever rephrasing or novel question has been overlooked. In this belief we bonded. With this mantra he could convince coauthors, often successfully, to pursue out-sized ambitions. For this inspiration he will be greatly missed.

Our collaboration lasted decades and pursued the most outrageous directions. We are graph theorists and combinatorialists by training, but Hunter would famously follow his astute instincts wherever they led, often into mathematical subjects we could never master in the short time we allotted to them. We learned a lot of hard lessons and good mathematics this way. Despite the lack of formal training in certain areas, Hunter would find gems wherever he sought them, often stirring up new vistas in otherwise calcified research areas. He had a knack for asking simple, yet provocative questions. We have so many half-written manuscripts (in number theory, morphisms on words, graph dynamics, and algebraic set theory, to name a few areas in which we dabbled). I am a slow writer, Hunter even slower. It saddens me to realize that many of Hunter's wildest ideas will now never be published. He was most proud of his imagination and the enthusiasm it evoked.

Enthusiasm gave us energy, but sometimes it caused trouble. In March of 2001 we became so excited about an apparent proof we had found of the Graceful Tree conjecture (applying the combinatorial nullstellensatz) that we couldn't calm down enough to verify it. We were both skeptical of the proof. I was in Kentucky and Hunter in Idaho, but the excitement surpassed such distances easily and prevented clear thinking. After a two-week time-out, we independently (and within hours of each other) found our error (the conjecture is still open). In an unusual fit of maturity, driven by the need to publish, we salvaged enough to produce a publishable paper [21]. A residue of our reckless enthusiasm still haunts that paper: Conjecture 2.4 is embarrassingly false, the subject of another half-written work that we never submitted. I can only hope the readers of that paper will forgive one (and only one!) poorly vetted conjecture.

During our friendship, Hunter and I often reminded each other of Hardy and Littlewood's *Four Axioms for Collaboration*, especially the second axiom: "when one received a letter from the other, he was under no obligation whatsoever to read it, let alone answer it" Each of us often struggled to get the other's attention. Back in the late 90's, Hunter pressed me to work on an extremal set theory problem, an area I have steadfastly avoided over the years because my intuition is so poor there. I pressed him to work on a problem characterizing imperfect graphs. He refused. We were at an impasse. Finally I reluctantly relented to work on his problem which, after he described it to me, I recognized as an old problem we'd unsuccessfully tried back in graduate school. Somewhat angry, I attempted to convince him to drop the problem by showing that every attack he proposed could not work. In his usual cavalier manner, he met my objections with dismissive laughter and a seemingly endless stream of new attacks (I contributed a few attacks too, but nowhere near as many he made). Indeed the harder I tried to deter him, the more stubbornly creative he became. Finally, six months after we joined our efforts, the well of new ideas went dry. We were exhausted and had made no progress. An eerie quiet fell upon our collaboration. I thought that the strain of this problem had broken our friendship. We didn't communicate for almost nine months. Then one day he called me and said triumphantly, I solved it! He had indeed done the best work of his career [36]. Because he'd struggled mightily with this problem since graduate school, he was most proud of this work. I would like to think I contributed to his efforts, but the ideas in that paper are entirely his. Fortunately for me, our friendship thrived. I was proud to write him a letter, when he asked me, supporting an award nomination for this impressive work.

I owe Hunter much. He contributed passionately to the open problems workshops each summer. He and his wife played matchmaker introducing me to my wife. We vacationed together at his summer home in Durango. He invited me to Idaho many summers to work. The best work of my career is the result of a question he asked [22]. All this pales in comparison to the enthusiasm and excitement he brought to the mathematical hunt he so enjoined. He was a gambler, figuratively and literally, and he enjoined large creative gambits in his pursuits. I try to conjure an image of him whenever my morale flags. He is missed.

Hunter Snevily: a Mentor, a Colleague, and a Friend *by Tanbir Ahmed*

In the summer of 2010, when I was a first year Computer Science Ph.D. student at Concordia, I received an email from Hunter Snevily showing interest to collaborate. Until then, my main achievements were proving some specific hard cases of problems in Ramsey Theory on the Integers using computer programs. I was excited to know that mathematicians were getting interested in my work. After a couple of emails, I realized that he was unwell and he would not be able to work much, but he would definitely guide me as required. Guidance soon became hours of short email conversations almost daily. He identified me as an Experimental Mathematician and encouraged me towards computer assisted proofs. The master of conjecture started to show me how to expand my horizons and find a career goal in the intersection of mathematics and computer science.

Hunter indulged me into online collaboration, taught me to ask questions to generate data, ask more questions based on the generated data, formulate conjectures (if possible) based on patterns, and try to prove them. We have worked on several problems in areas like Ramsey Theory on the Integers, Permutations, and Graph Theory, and also in areas like Discrete Geometry which was previously unexplored by both of us. Every assignment has been a fruitful learning experience for me. His observation of some surprising patterns in the data contained in my previous papers led us to an experimental paper on the van der Waerden numbers $w(2;3,t)$ [4]. In [3], we have generalized a well known conjecture of Szekeres on the length of the longest k-AP free subsequences in $1,2,\ldots,n$, namely, $r(k,n)$. Hunter guided me towards the upper bound $r(k,n) \leqslant n - \lfloor m/2 \rfloor$ where $n = m(k-1)+t$ and $t < k-1$. In [1], we have generated computational data regarding Erdős and Fishburn's conjecture which says that every $g(k)$ (maximum number of points on the Euclidean plane that contain exactly k distances) point subset of the plane that determines k different distances is similar to a subset of the triangular lattice. We have provided a method of construction that unifies known optimal point configurations for $k \geqslant 3$. This is a nice example of an application of experimental methods to make some progress on a hard problem. Hunter was an avid supporter of experimental mathematics and this was the common ground where we bonded.

Apart from mathematics, Hunter liked to share his opinions about life and would not hesitate to offer a piece of advice whenever he would feel it necessary. Without any reservation in mind, he shared with me the letter he wrote for his children Madison and John. He was possibly going through the worst phase of his struggle with Parkinson's disease yet preserving his apparently never-ending enthusiasm and quest for conjectures. He often used to warn "finish the work, I won't be around forever". I often provided him some new data with a hope that he would formulate a conjecture, have some joy of mathematics, and continue his fight to live. I am not sure whether this trick worked or not but a couple of days without his emails would certainly make me worried with frightening thoughts. Our collaboration continued for three years leading towards several (six to be precise) publications amidst joy of discovery mixed with a fear of an imminent unfortunate loss. On November 11, 2013, the dreaded news arrived, and I experienced for the first time how it feels to lose a mentor, a colleague, and such a good friend. To me, he was like an angel who came out of nowhere when I needed the most. He is deeply missed and will be fondly remembered.

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